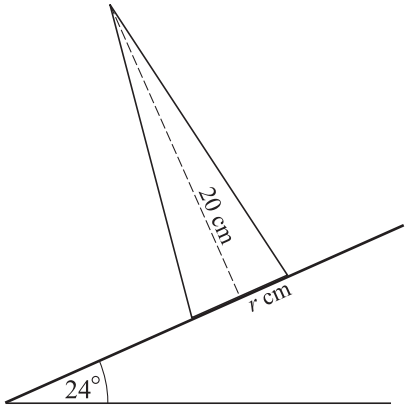
OCR Maths M2

Topic Questions from Papers

Centre of Mass

1



A uniform solid cone has vertical height 20 cm and base radius r cm. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cone topples when the angle of inclination is 24° (see diagram).

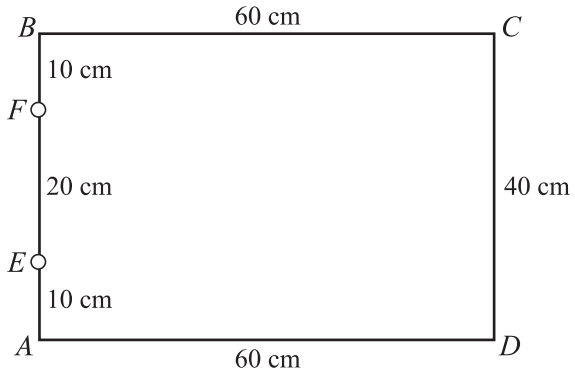
(i) Find r, correct to 1 decimal place. [4]

A uniform solid cone of vertical height 20 cm and base radius 2.5 cm is placed on the plane which is inclined at an angle of 24°.

(ii) State, with justification, whether this cone will topple. [1]

(Q1, June 2005)

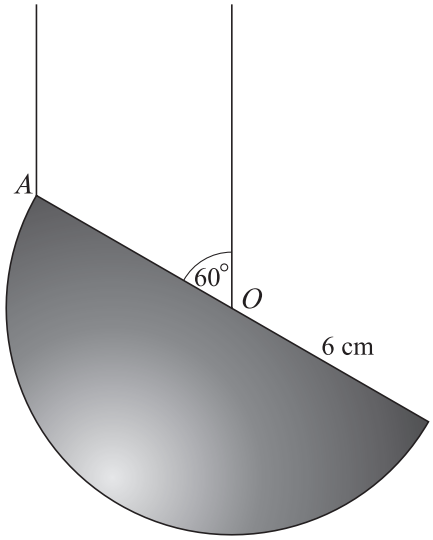
2



(iii) Calculate the distance from AD of the centre of mass of the frame. [3]

(Q4, Jan 2006)

3



A uniform solid hemisphere of weight 12 N and radius 6 cm is suspended by two vertical strings. One string is attached to the point O, the centre of the plane face, and the other string is attached to the point A on the rim of the plane face. The hemisphere hangs in equilibrium and OA makes an angle of 60° with the vertical (see diagram).

(i) Find the horizontal distance from the centre of mass of the hemisphere to the vertical through O. [2]

(ii) Calculate the tensions in the strings. [5]

(Q3, June 2006)

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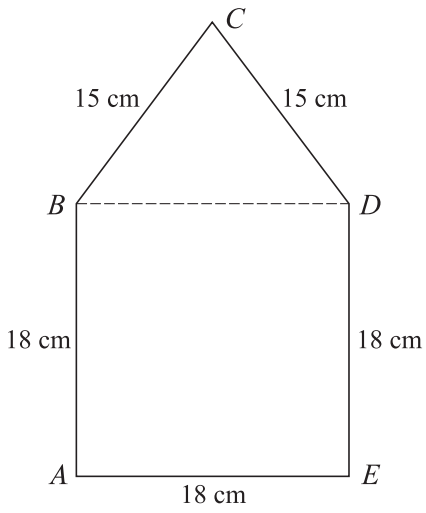

A uniform lamina $ABCDE$ consists of a square and an isosceles triangle. The square has sides of 18 cm and $BC = CD = 15$ cm (see diagram).

(i) Taking x- and y-axes along AE and AB respectively, find the coordinates of the centre of mass of the lamina. [7]

(ii) The lamina is freely suspended from B. Calculate the angle that BD makes with the vertical. [2]

(Q5, June 2006)

5 A uniform solid cylinder has height 20 cm and diameter 12 cm. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cylinder topples when the angle of inclination is α. Find α. [3]

(Q1, Jan 2007)

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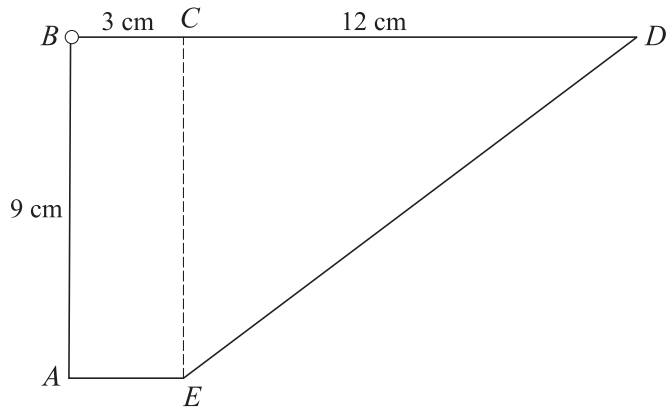

A uniform lamina $ABCDE$ of weight 30 N consists of a rectangle and a right-angled triangle. The dimensions are as shown in the diagram.

(i) Taking x- and y-axes along AE and AB respectively, find the coordinates of the centre of mass of the lamina. [8]

The lamina is freely suspended from a hinge at B.

(ii) Calculate the angle that AB makes with the vertical. [2]

The lamina is now held in a position such that BD is horizontal. This is achieved by means of a string attached to D and to a fixed point 15 cm directly above the hinge at B.

(iii) Calculate the tension in the string. [3]

(Q6, Jan 2007)

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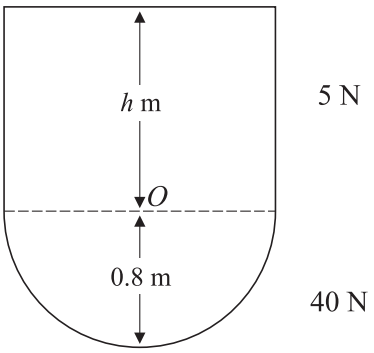

Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height h m. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point O (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from O.

(i) Show that $h = 1.2$. [6]

(Q8, June 2007)

8 **(i)** A uniform semicircular lamina has radius 4 cm. Show that the distance from its centre to its centre of mass is 1.70 cm, correct to 3 significant figures. [2]

(ii)

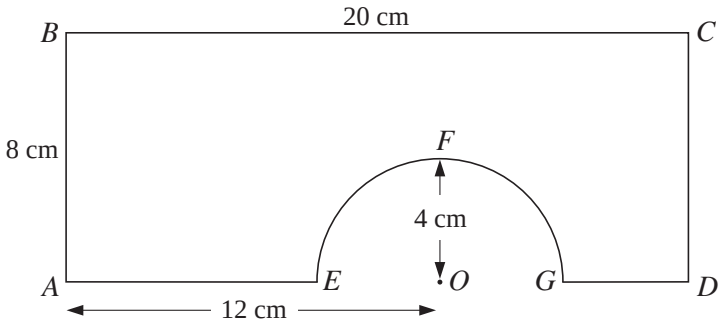

Fig. 1

A model bridge is made from a uniform rectangular board, *ABCD*, with a semicircular section, *EFG*, removed. *O* is the mid-point of *EG*. *AB* = 8 cm, *BC* = 20 cm, *AO* = 12 cm and the radius of the semicircle is 4 cm (see Fig. 1).

(a) Show that the distance from *AB* to the centre of mass of the model is 9.63 cm, correct to 3 significant figures. [5]

(b) Calculate the distance from *AD* to the centre of mass of the model. [4]

(Q8, Jan 2008)

9

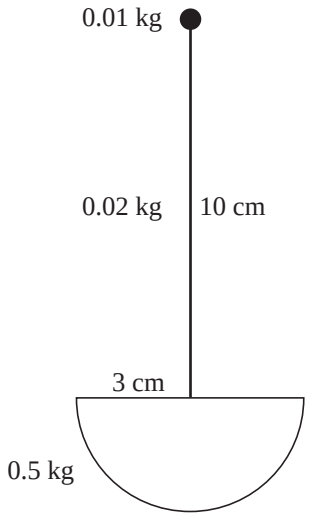

Fig. 1

A toy is constructed by attaching a small ball of mass 0.01 kg to one end of a uniform rod of length 10 cm whose other end is attached to the centre of the plane face of a uniform solid hemisphere with radius 3 cm. The rod has mass 0.02 kg, the hemisphere has mass 0.5 kg and the rod is perpendicular to the plane face of the hemisphere (see Fig. 1).

(i) Show that the distance from the ball to the centre of mass of the toy is 10.7 cm, correct to 1 decimal place. [4]

(Q5, June 2008)

10 **(i)**

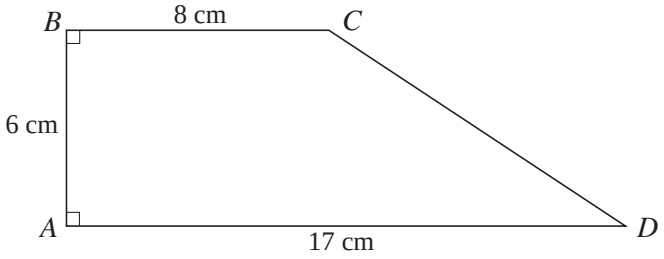

Fig. 1

A uniform lamina $ABCD$ is in the form of a right-angled trapezium. $AB = 6$ cm, $BC = 8$ cm and $AD = 17$ cm (see Fig. 1). Taking x- and y-axes along AD and AB respectively, find the coordinates of the centre of mass of the lamina. [8]

(Q8, June 2008)

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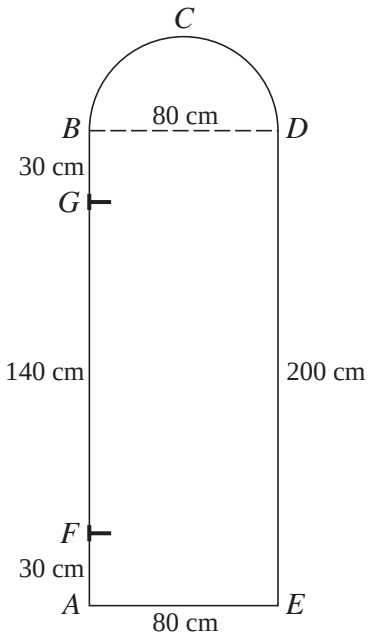

A door is modelled as a lamina $ABCDE$ consisting of a uniform rectangular section $ABDE$ of weight 60 N and a uniform semicircular section BCD of weight 10 N and radius 40 cm. AB is 200 cm and AE is 80 cm. The door is freely hinged at F and G, where G is 30 cm below B and F is 30 cm above A (see diagram).

(ii) Calculate the distance from AE to the centre of mass of the door. **[6]**

(Q3, Jan 2009)

12 **(i)**

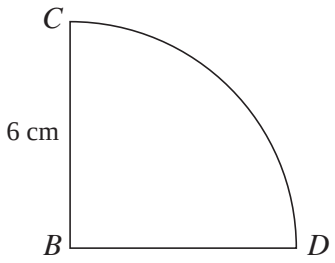

Fig. 1

Fig. 1 shows a uniform lamina *BCD* in the shape of a quarter circle of radius 6 cm. Show that the distance of the centre of mass of the lamina from *B* is 3.60 cm, correct to 3 significant figures. **[2]**

A uniform rectangular lamina *ABDE* has dimensions *AB* = 12 cm and *AE* = 6 cm. A single plane object is formed by attaching the rectangular lamina to the lamina *BCD* along *BD* (see Fig. 2). The mass of *ABDE* is 3 kg and the mass of *BCD* is 2 kg.

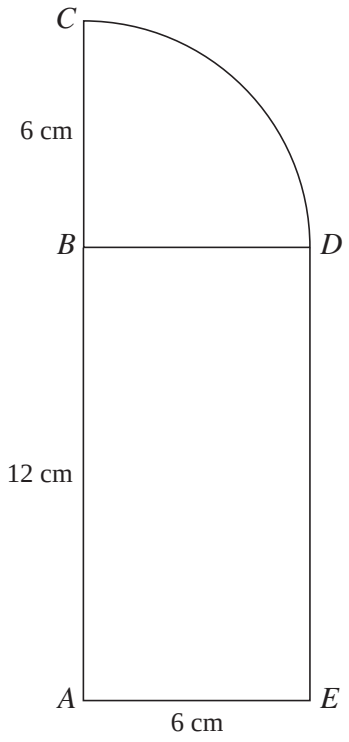

Fig. 2

(ii) Taking *x*- and *y*-axes along *AE* and *AB* respectively, find the coordinates of the centre of mass of the object. **[7]**

The object is freely suspended at *C* and rests in equilibrium.

(iii) Calculate the angle that *AC* makes with the vertical. **[2]**

(Q5, June 2009)

13

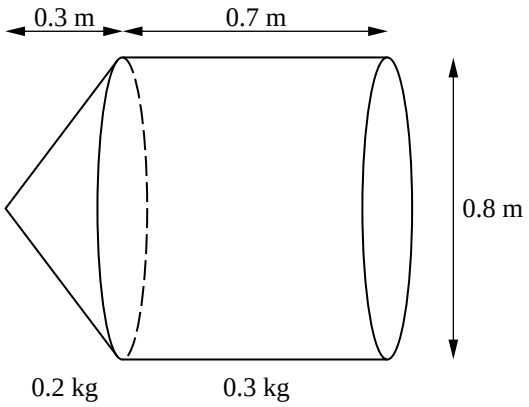

Fig. 1

A uniform conical shell has mass 0.2 kg, height 0.3 m and base diameter 0.8 m. A uniform hollow cylinder has mass 0.3 kg, length 0.7 m and diameter 0.8 m. The conical shell is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

(i) Show that the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m. **[4]**

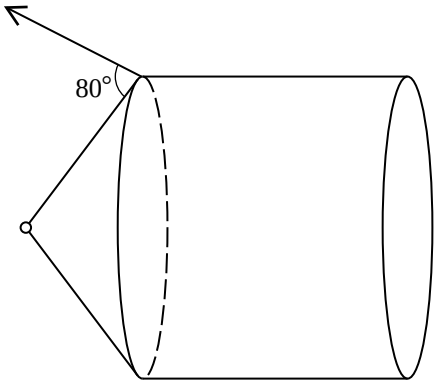

Fig. 2

The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of 80° with the slant edge of the conical shell (see Fig. 2).

(ii) Calculate the tension in the wire. **[4]**

(Q3, Jan 2010)

14 **(i)**

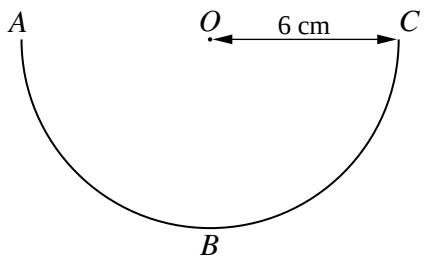

Fig. 1

A uniform piece of wire, *ABC*, forms a semicircular arc of radius 6 cm. *O* is the mid-point of *AC* (see Fig. 1). Show that the distance from *O* to the centre of mass of the wire is 3.82 cm, correct to 3 significant figures. **[2]**

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(ii)

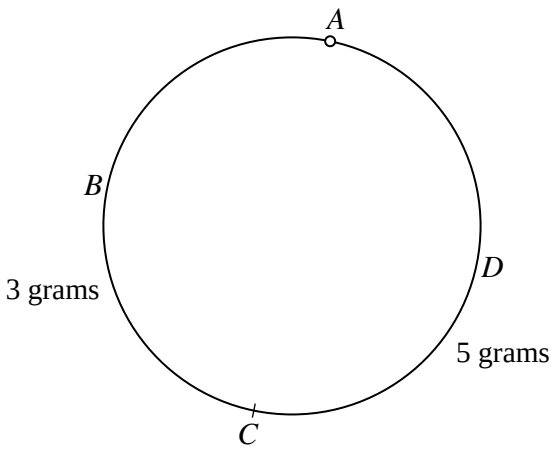

Fig. 2

Two semicircular pieces of wire, *ABC* and *ADC*, are joined together at their ends to form a circular hoop of radius 6 cm. The mass of *ABC* is 3 grams and the mass of *ADC* is 5 grams. The hoop is freely suspended from *A* (see Fig. 2). Calculate the angle which the diameter *AC* makes with the vertical, giving your answer correct to the nearest degree. **[5]**

(Q2, June 2010)

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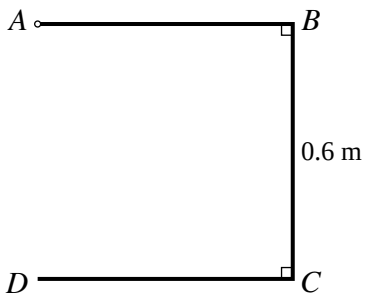

A uniform square frame *ABCD* has sides of length 0.6 m. The side *AD* is removed from the frame, and the open frame *ABCD* is attached at *A* to a fixed point (see diagram).

(i) Calculate the distance of the centre of mass of the open frame from *A*. **[5]**

(Q1, Jan 2011)

16 A uniform solid is made of a hemisphere with centre *O* and radius 0.6 m, and a cylinder of radius 0.6 m and height 0.6 m. The plane face of the hemisphere and a plane face of the cylinder coincide. (The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.)

(i) Show that the distance of the centre of mass of the solid from *O* is 0.09 m. **[5]**

(Q5, Jan 2011)

17 A uniform lamina *ABCDE* consists of a square *ACDE* and an equilateral triangle *ABC* which are joined along their common edge *AC* to form a pentagon whose sides are each 8 cm in length.

(i) Calculate the distance of the centre of mass of the lamina from *AC*. **[5]**

(ii) The lamina is freely suspended from *A* and hangs in equilibrium. Calculate the angle that *AC* makes with the vertical. **[2]**

(Q3, June 2011)

18

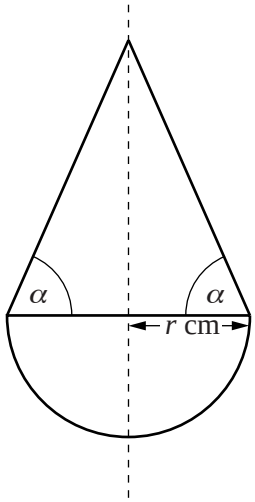

Fig. 1

A child's toy is a uniform solid consisting of a hemisphere of radius r cm joined to a cone of base radius r cm. The curved surface of the cone makes an angle α with its base. The two shapes are joined at the plane faces with their circumferences coinciding (see Fig. 1). The distance of the centre of mass of the toy above the common circular plane face is x cm.

[The volume of a sphere is $\frac{4}{3}\pi r^3$ and the volume of a cone is $\frac{1}{3}\pi r^2 h$.]

(i) Show that $x = \dfrac{r(\tan^2\alpha - 3)}{8 + 4\tan\alpha}$. **[4]**

The toy is placed on a horizontal surface with the hemisphere in contact with the surface. The toy is released from rest from the position in which the common plane circular face is vertical (see Fig. 2).

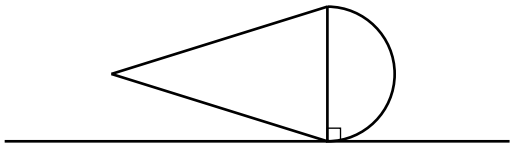

Fig. 2

(ii) Find the set of values of α such that the toy moves to the upright position. **[3]**

(Q2, Jan 2012)

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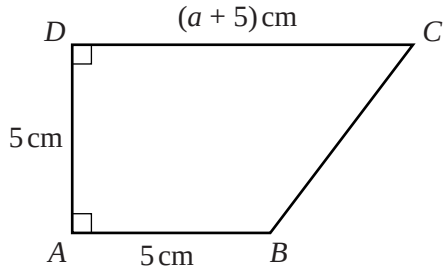

The diagram shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a trapezium $ABCD$ with AB and CD perpendicular to AD. The lengths of AB and AD are each 5 cm and the length of CD is $(a + 5)$ cm.

(i) Show the distance of the centre of mass of the prism from AD is

$$\frac{a^2 + 15a + 75}{3(a + 10)} \text{ cm}.$$ **[5]**

The prism is placed with the face containing AB in contact with a horizontal surface.

(ii) Find the greatest value of a for which the prism does not topple. **[3]**

The prism is now placed on an inclined plane which makes an angle $\theta°$ with the horizontal. AB lies along a line of greatest slope with B higher than A.

(iii) Using the value for a found in part **(ii)**, and assuming the prism does not slip down the plane, find the greatest value of θ for which the prism does not topple. **[6]**

(Q7, June 2012)

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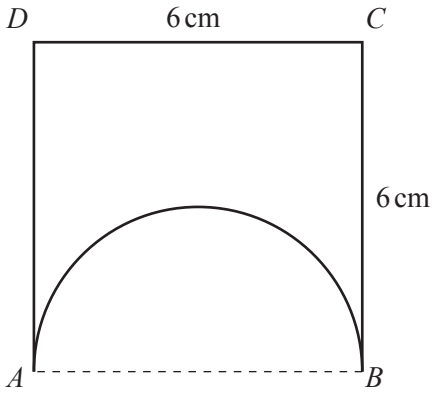


A uniform square lamina $ABCD$ of side 6 cm has a semicircular piece, with AB as diameter, removed (see diagram).

(i) Find the distance of the centre of mass of the remaining shape from CD. **[6]**

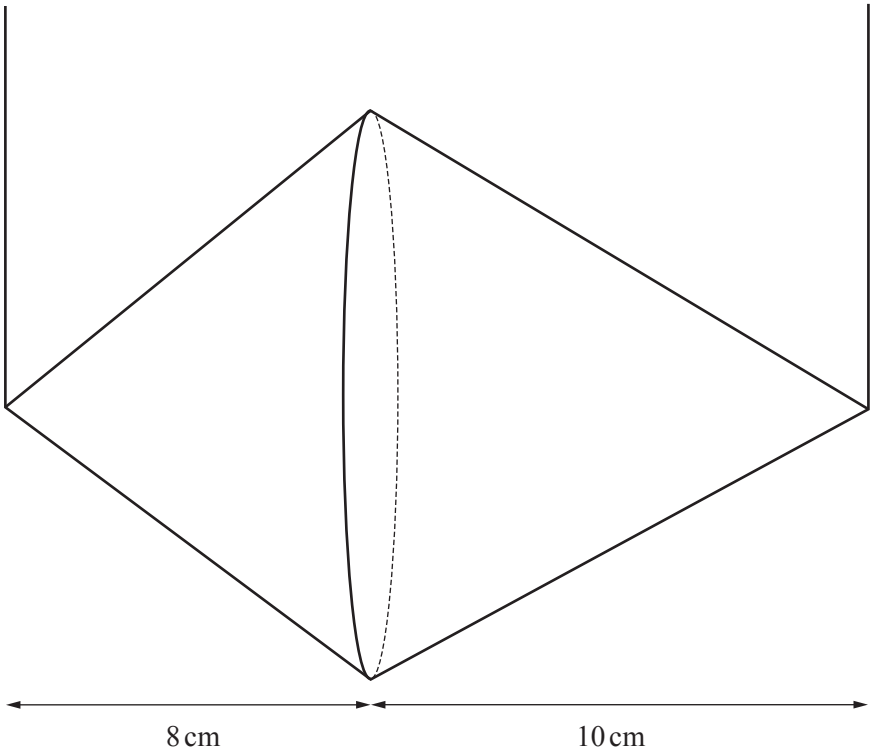
The remaining shape is suspended from a fixed point by a string attached at C and hangs in equilibrium.

(ii) Find the angle between CD and the vertical. **[2]**

(Q4, Jan 2013)

21 A solid uniform cone has height 8 cm, base radius 5 cm and mass 4 kg. A uniform conical shell has height 10 cm, base radius 5 cm and mass 0.4 kg. The two shapes are joined together so that the circumferences of their circular bases coincide.

(i) Find the distance of the centre of mass of the shape from the common circular base. **[4]**



The object is suspended with a string attached to the vertex of the cone and another string attached to the vertex of the conical shell. The object is in equilibrium with the strings vertical and the axis of symmetry of the object horizontal (see diagram).

(ii) Find the tension in each string. **[4]**

(Q4, June 2013)